



# Designing life levels of Extreme Temperature by 2100

International Meeting on Statistical Climatology

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## Climate Change

- Increase in **frequency and intensity** of extremely hot events.[1]
- Increasing knowledge of the warming phenomena, using both **observations and climate models**.

## Safety concerns

- Reliability of **safety-significant equipment**.
- Building Codes using stationary return levels which may **vary during the building's life**.

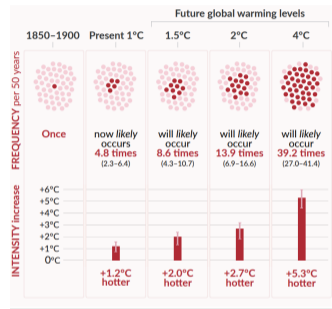


Figure: IPCC, 2021: Summary for Policymakers by Masson Delmotte, V et al.[4]

**Our Goal:** Defining the risk of extreme temperature levels excess by 2100 at a local scale.

How?

- Adapting the stationary return level to a **non-stationary context**, considering the lifetime of the building.
- Estimating extreme temperature levels, integrating **information from climate models and local observations**, using tools based on Extreme Value Theory and a Bayesian framework.
- Providing a usable estimate taking **uncertainty** into account.
- Applying the method to a **place of interest**, Tricastin (South of France).

Need:

- Separating the **period of interest** from the **return period** (annual probability  $p = \frac{1}{T}$ ).
- Account for **non-stationnarity**,  $Y_{2023} \neq Y_{2050}$ .
- Applied similarly with or without stationarity.

Our choice: Equivalent Reliability[5] [3].

For period  $t_1 : t_2$ , solution  $\mathbf{z}_p$  of :

$$P[\max(Y_{t_1}, Y_{t_1+1}, \dots, Y_{t_2}) \leq \mathbf{z}_p] = (1 - p)^{t_2 - t_1 + 1}$$

Constraints:

- **Extreme Values Analysis:** Annual Maxima, use of **GEV** distribution.
- **Non-stationarity:** Mean European Temperature as covariate for relationship with time and **scenario integration**.

$$Y \sim GEV(\mu_t, \sigma_t, \xi)$$

$$\begin{cases} \mu(t) &= \mu_0 + \mu_1 X_t \\ \sigma(t) &= \exp(\sigma_0 + \sigma_1 X_t) \\ \xi(t) &= \xi_0 \end{cases}$$

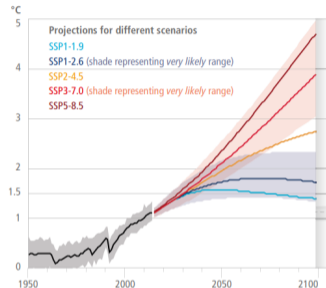
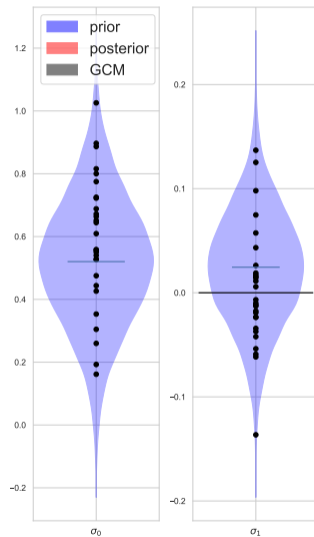


Figure: Global surface temperature changes for various scenarios. ( IPCC, 2022)

## Bayesian framework[6]

### A-priori knowledge

- Include only information from **climate models**.  
(historical and scenario).



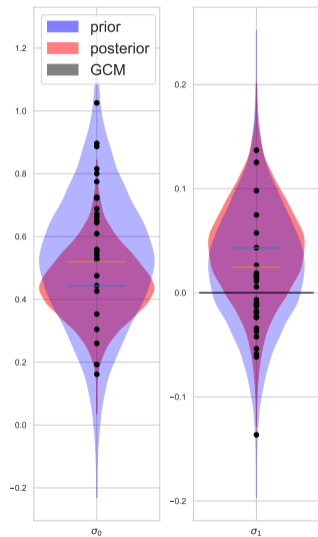
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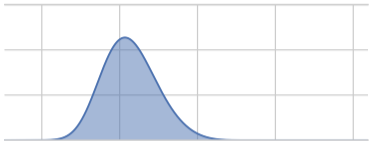
### A-priori knowledge

- Include only information from **climate models**. (historical and scenario).

### Updated using observations

- Maxima constraint using **Markov chain Monte Carlo** (NUTS).
- Using past local observations.

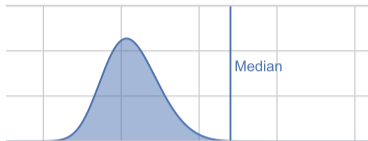




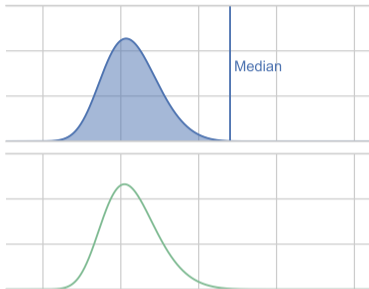
- Using all draws: for return levels **median**

■ Median distribution    ■ Predictive distribution



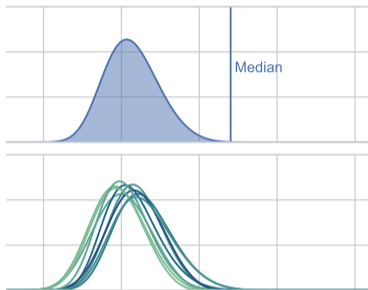


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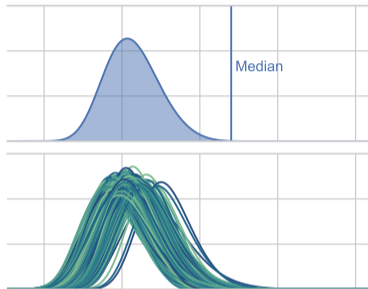
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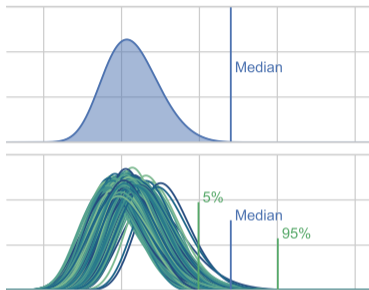
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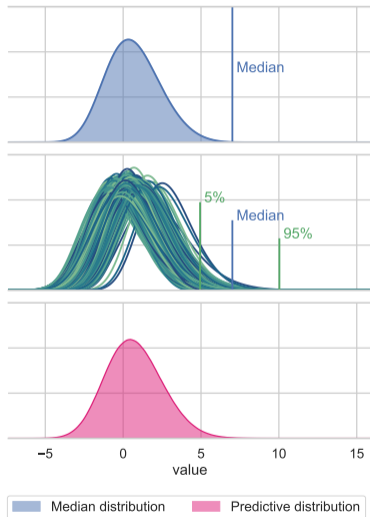
- Using all draws: for return levels **median** and **confidence intervals**

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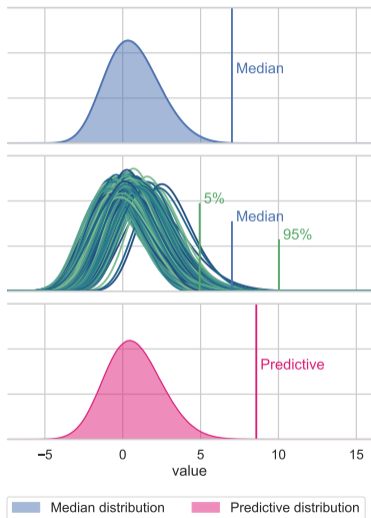
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- **Predictive distribution[2]**: One distribution **averaged** over the distribution of the model parameters.

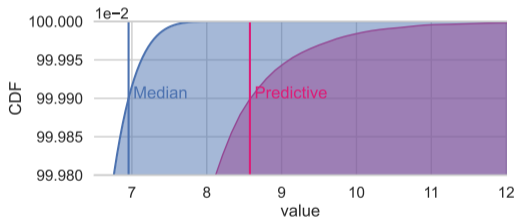
$$P(Z \leq z|z_0) = \int_{\Theta} P(Z \leq z|\theta)\pi(\theta|z_0)d\theta$$



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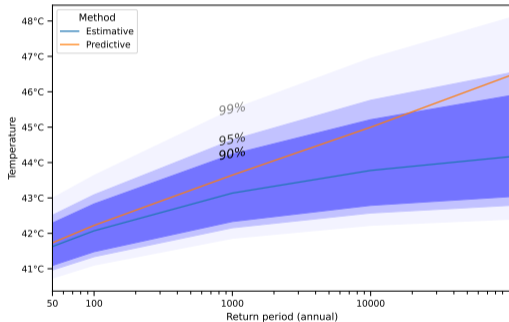


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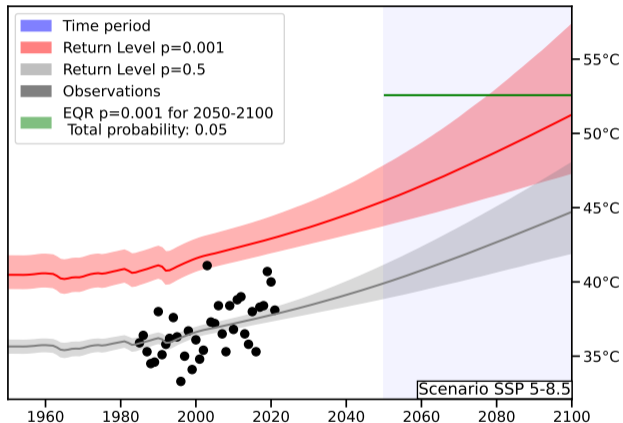




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Period of interest: **2050-2100**.

For an equivalent return level of 1000 years:

- Predictive is 52.9°C
- Median is 50.3°C with 55.5°C for 95% upper bound.

### Interpretation

53°C has an annual probability of excess of  $\frac{1}{1000}$  over 2050-2100. Similarly, 53°C has a **5% probability of excess over 2050-2100**.

- Chose **Equivalent Reliability** as quantity of interest.
- Adapted Robin and Ribes' (2020) estimation method.
- MCMC Algorithm: Large improvement in **time and precision**, comparison with alternatives.
- **Predictive** estimation taking parameter **uncertainty** into account.
- **First Application** at Tricastin.
- Many potential **avenues of improvement** (Data, Prior specification, model specification, etc).

- [1] Chapter 11: Weather and Climate Extreme Events in a Changing Climate.
- [2] S. Coles and J. Tawn.  
Bayesian modelling of extreme surges on the UK east coast.  
*Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 363(1831):1387–1406, June 2005.  
Publisher: Royal Society.
- [3] Y. Hu, Z. Liang, V. P. Singh, X. Zhang, J. Wang, B. Li, and H. Wang.  
Concept of Equivalent Reliability for Estimating the Design Flood under Non-stationary Conditions.  
*Water Resources Management*, 32(3):997–1011, Feb. 2018.

- [4] Intergovernmental Panel On Climate Change (Ipcc).  
*Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change.*  
Cambridge University Press, 1 edition, July 2023.
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Study on the estimation of design value under non-stationary environment.  
*South-to-North Water Transfers Water Sci Tech*, 14:50–53, 2016.
- [6] Y. Robin and A. Ribes.  
Nonstationary extreme value analysis for event attribution combining climate models and observations.  
*Advances in Statistical Climatology, Meteorology and Oceanography*, 6(2):205–221,  
Nov. 2020.  
Publisher: Copernicus GmbH.



# Designing life levels of Extreme Temperature by 2100

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*Thanks for listening.  
Any questions?*

Plan

■ Supplementary



Only need to be able to simulate from conditionnal distributions. (Maybe possible use of  $X_T$ )

Multivariate :  $\psi = (\psi_1, \dots, \psi_d)'$  , full conditionals are  $\pi(\psi_i | \psi_{-i}) = \pi_i(\psi_i)$

Description of algorithm:

- Initialisation:  $k=1$ , initial state of chain  $\psi^{(0)}$
- Boucle: For new value  $\psi^{(k)}$  :
  - $\psi_1^{(k)} \sim \pi(\psi_1 | \psi_{-1}^{(k-1)})$
  - $\psi_2^{(k)} \sim \pi(\psi_2 | \psi_{-1,2}^{(k-1)}, \psi_1^{(k)})$
  - ...
  - $\psi_d^{(k)} \sim \pi(\psi_d | \psi_{-d}^{(k)})$

$\pi(\psi)$  is still the density of interest. We now have a transition kernel  $p(\psi_{i+1}, \psi_i)$ , easy to simulate from, to get successive values.

- Initialisation :  $k=1$ , initial state of chain  $\psi^{(0)}$
- Boucle: For new value  $\psi^{(k)}$  :
  - Generate new proposed value  $\psi'$  using the kernel transition function.
  - Calculate Acceptance Probability (ratio)  $A(\psi^{(k-1)}, \psi')$  of the proposed change of value:

$$A(\psi^{(k)}, \psi') = \min\left\{1, \frac{\pi(\psi')L(\psi'|\mathbf{x})p(\psi', \psi^{(k-1)})}{\pi(\psi^{(k-1)})L(\psi^{(k-1)}|\mathbf{x})p(\psi^{(k-1)}, \psi')}\right\}$$

- Accept  $\psi^{(k)} = \psi'$  with probability  $A(\psi^{(k)}, \psi')$  and keep  $\psi^{(k)} = \psi^{(k-1)}$  otherwise.

Based on Bayesian Modelling of Extreme Rainfall Data from Elizabeth Smith  
Gibbs concept (each parameter is updated in turn) and conditionals are MH (do we accept the new value produced by the transition function?)

Avantage: Each parameter has his own trajectory (One may not move much and another a lot) + varying transition kernel (proportionnal) (not hard to do for simple MH too)

→ Less dependance than normal MH?.

Description of algorithm:

- Initialisation :  $k=1$ , initial state of chain  $\theta^{(0)}$
- Boucle: For new value  $\theta^{(k)}$  :
  - In turn, for each parameter  $\theta_j^{(k)}$ 
    - $\theta'_j = \theta_j^{(k-1)} + \varepsilon_j$
    - Accept or refuse using  $A(\theta_j^{(k-1)}, \theta'_j)$  with  $\theta_{-j}^{(k)}$  seen as known.